Analysis of Descriptor Systems for Control Applications using SCILAB

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***Resumen*— In this paper we present the summary of a series of methodologies for the analysis of descriptor systems in the open source software SCILAB. The properties of regularity, solvability, C-controllability (observability), R-controllability (observability), I-controllability and stability are analyzed. The Karampetakis method for discretized continuous systems in descriptor form is presented and an algorithm of solution is proposed. Unlike the state-space systems, in non-singulars determining these properties is not always a trivial task, and on occasion can be mean a difficult task because there is not a specialized toolbox. Therefore, in this paper we propose a series of functions that complement those of SCILAB to analyze descriptor systems and can be used in control applications such as design of observers or fault detection systems. © UNAM-AMCA.**

Keys words: Descriptor systems, Scilab, C-Controllability, C-Observability, Discretization of Descriptor systems, Singular systems.

I. Introduction

Linear Time Invariant Descriptor System have the form

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$
$$y(t) = Cx(t) + Du(t) \tag{2}$$

for continuous time

$$E\dot{x}(t+1) = Ax(t) + Bu(t) \tag{3}$$
$$y(t) = Cx(t) + Du(t) \tag{4}$$

for discrete, with $t \in \mathbb{Z}$, where $A, E \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$. E is singular and not necessarily diagonal, the behavior relationships are not necessarily dynamics.

The main difference between typical state-space systems and descriptors systems, is that the matrix E of (1-3) is singular and therefore not invertible, because of this descriptors method must meet certain conditions for its analysis and management in the design of systems for monitoring and control.

Two of the most important properties that descriptors systems must meet are solvability and regularity. If these properties hold, a singular system can be converted to an equivalent form and investigate other properties such as controllability, observability and stability. If the system does not meet with the regularly property, then the system must be regularized.

Property of regularity is related to the invertibility of matrix pencil pair $(E,\, A)$,in (Gantmacher, 1959) is defined as:

Definition 1: The system (1) is called regular if there exists a constant scalar γ such that $\det(\gamma E - A) \neq 0$ or equivalently, the polynomial $\det(sE - A)$ is not identically zero. In this case, we also say that the matrix pair $(E,\, A)$, or the matrix pencil $sE - A$; is regular.
Solvability should be defined as the existence of a unique solution for any given sufficiently differentiable $u(t)$and any given admissible initial condition corresponding to the given $u(t)$.

Definition 2: $(A,\, E)$ is solvable if the matrix pencil $E + \lambda A$ is regular. In other words the matricial relationship (E, A) is solvable if and only if $|sE - A| \neq 0$, or equivalently if there exist an scalar $\lambda \in \mathbb{C}$ such that $|\lambda E - A| \neq 0$.where $|\centerdot|$ is the determinant (Yip y Sincovec, 1981).

Other important property related with the regularity of the pair pencil $(E,\, A)$ is the existence of an equivalent form. This forms are know like Restricted Systems Equivalent Forms or simply r.s.e forms.

Theorem 1: We call two systems $(E;\, A,\, B,\, C,\, D)$ and $(\hat{E};\, \hat{A},\, \hat{B},\, \hat{C},\, \hat{D})$ restricted system equivalent (r.s.e.) if their order, number of inputs and outputs are equal and there exist two non singular matrices P and Q such that $\hat{E} = PEQ$, $\hat{A} = PAQ$, $\hat{B} = PB$, $\hat{B} = CQ$, $\hat{D} = D$ (Voigt, 2010; Gantmacher, 1959). Where

$$PEQ = \begin{bmatrix} I_n & 0 \\ 0 & N \end{bmatrix} \tag{5}$$

$$PAQ = \begin{bmatrix} A_1 & 0 \\ 0 & I_r \end{bmatrix} \tag{6}$$

when $A_1 \in \mathbb{R}^{n_1\times n_1}$ is a matrix whose elements are the finite eigenvalues, $I_k \in \mathbb{R}^{k\times k}$ is the identity matrix and $N \in \mathbb{R}^{k\times k}$ is a nilpotent matrix also in Jordan form. A_1 and N are unique up to permutation of Jordan blocks .

The equivalent system is (Sokolov, 2006):

$$\dot{x_1} = A_1 x_1(t) + B_1 u(t) \tag{7}$$
$$y_1(t) = C_1 x_1(t)$$
$$N x_2 = x_2(t) + B_2 u(t) \tag{8}$$
$$y_2(t) = C_2 x_2(t)$$
$$y = y_1(t) + y_2(t) = C_1 x_1(t) + C_2 x_2(t) \tag{9}$$

where:

$$\left[\begin{array}{c} x_1(t) \\ x_2(t) \end{array}\right] = Q^{-1}x(t), \quad \left[\begin{array}{c} B_1 \\ B_2 \end{array}\right] = PB, \quad [C_1\, C_2] = CQ$$

Subsystem (7) represents the *slow response* and subsystem (8) represents the *fast response*

A. Laurent expansion coefficients

Considering a regular descriptor system in the form of (1) , then the Laurent Expansion of $(sE - A)^{-1}$ is (Karampetakis, 2003):

$$\begin{aligned}\Phi(s) &= (sE-A)^{-1} = \Phi_{-k}s^{k-1} + \cdots + \Phi_{-2}s + \Phi_0 s^0 \\ &\quad +\Phi_1 s^{-1} + \ldots \Phi_k s^{-k-1} \qquad (10)\\ &= \sum_{k=h}^{\infty} \Phi_k\left(E,A\right)s^{-k-1}\end{aligned}$$

whit $k = 1, 2, ...h$, where h is the nilpotence index of $(sE - A)$, and Φ are the fundamental matrix defined in (Lewis, 1985; Lewis, 1990). The fundamental matrices Φ_0 and Φ_{-1} are obtained by the Drazin inverse of the matrix A (Bernstein, 2009; Stykel, 2006; Ji, 2002):

$$A^D = S \begin{bmatrix} J_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} S^{-1} \tag{11}$$

In

this is modified using the the transformations matrix P and Q, then :

$$\Phi_k = P\left[\begin{array}{cc} A_1^k & 0 \\ 0 & 0 \end{array}\right] Q \;\; k \geq 0 \tag{12}$$
$$\Phi_{-1} = P\left[\begin{array}{cc} 0 & 0 \\ 0 & -N^{k-1} \end{array}\right] Q \;\; k > 0 \tag{13}$$

II. Properties of descriptor systems

1) Controllability and observability: In descriptors systems there exists principally three kinds of controllability this are: C-Controllability. C- Controllability is related with the fact that the system can controlled with any initial conditions. R-Controllability means that the system only could controlled by a reduced set of initial conditions and I-Controllability means that the system can control impulsive signals in the start up.

The controllability in Descriptor systems can be found of different ways, e.g. using Laurent expansion, r.s.e or direct form approaches. In Laurent expansion approach, the controllability is related with the dimension of (E, A) and rank of matrices Φ_0 and Φ_{-2}. In r.s.e approach the controllability can be found by the analysis of controllability of subsystems slow (7) and fast (8) like is analyzed in (Duan, 2010, cap. 4, pp. 131). While in direct form approach the controllability is found by analysis of matrices $(E,\, A,\, B)$ in singular form (see (Yip y Sincovec, 1981; Duan, 2010, cap. 4, pp. 156)).

Like controllability there exist different forms of observability (C- Observability, R-Observability and I-Observability). C- Observability means for any initial conditions, we can always observer the states. R-Observability meas that only by a reduced set of initial conditions the states can be observed. I-Observability is concerned with observing the impulse terms in the system state response from the output data of the system (Duan, 2010). Observability principally can be found in three different forms: Laurent expansion (Koumboulis y Mertzios, 1999), r.s.e (Duan, 2010, cap. 4, pp. 142) and direct approaches (Yip y Sincovec, 1981; Duan, 2010, cap. 4, pp. 157). This forms can be selected with the next function:

2) Stability: In typical state-space system we say that a system is stable if the eigenvalues of $(sE - A) \in \mathbb{C}^-$, for descriptor system the criterion is basically the same and is resumed in the next theorem:

Theorem 2: A regular descriptor system is stable if and only if

$$\mathsf{eig}(E,\, A) =\subset \mathbb{C}^- = \{s|\, s \in \mathbb{C},\, \mathsf{Re}(s) < 0\} \tag{14}$$

From the theorem is deducible that stability is defined be the position of the eigenvalues. If the system has negative real part then, is stable. If the system has zero real part then is critically stable and if has positive real part, then is unstable.

On the other hand, the stability can be determined using the following generalized Lyapunov equation (GLE)

$$E^T XA + A^T XE = -P_r^T Y P_r \tag{15}$$

for continuous system, and for discrete system:

$$A^T XA - E^T XE = -P_r^T Y P - (I - P_r)^T Y (I - P_r) \tag{16}$$

where P_l and P_r are the spectral projections onto the left and right finite deflating subspaces of the pencil $\lambda E - A$ along the left and right infinite deflating subspaces, respectively, then (Stykel, 2002)

$$P_l = Q^{-1}\begin{bmatrix} I_{n1} & 0 \\ 0 & 0 \end{bmatrix} Q \qquad P_r = Q\begin{bmatrix} I_{n1} & 0 \\ 0 & 0 \end{bmatrix} Q^{-1} \tag{17}$$

The deflating subspaces of $\lambda E - A$ corresponding to the finite (infinite) eigenvalues we will call the finite (infinite) deflating subspaces. Like a result of GLE (15) the next theorem is defined

Theorem 3: (Stykel, 2002) Let P_r be the spectral projection onto the right finite deflating subspace of a regular pair

- If there exist a positive definite matrix$Y$ and a positive semidefinite matrix X satisfying (15), then the matrix pair $(E,\, A)$ is stable.
- If the matrix pair $(E,\, A)$ is stable, then for every positive definite matrix Y , (15) has a positive semidefinite solution X .

Note that P_r can be computed using the results of command pencan, while the LMI's of the Theorem (3) can be programed using a LMI tool. Although scilab has its own tool to analyze LMI, in this work the SciYalmip was used (Lofberg, 2011).

A. Discretization

Using the Laurent expansion coefficients, the continuous system (1), can be rewritten as (Karampetakis, 2003):

$$\underbrace{\begin{bmatrix} \rho I_n - \Phi_0 A & 0 \\ 0 & I_n + \rho\Phi_{-1}E \end{bmatrix}}_{\rho\tilde{E}-A} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{\tilde{x}(t)} = \underbrace{\begin{bmatrix} \Phi_0 B \\ \Phi_{-1}B \end{bmatrix}}_{\tilde{B}} u(t) \tag{18}$$

then, instead of discretize system (1), we may discretize the system (18), giving rise to the following theorem:

Theorem 4: (Karampetakis, 2003) Using a zero-order hold approximation of the input$u(t)$ and first-order hold approximation of the derivatives of the input$u(t)$, the continuous time nonhomogeneous singular system is discretized to yield the singular state space system:

$$\left\{ \begin{array}{rcl} x_1((k+1)T) & = & \tilde{A}x_1(kT) + \tilde{B}_1 u(kT) \\ \tilde{E}_1 x_2((k+1)T) & = & x_2(kT) + \tilde{B}_2 u(kT) \end{array} \right\} \tag{19}$$

$$x(kT) = \begin{bmatrix} I_n & I_n \end{bmatrix} \begin{bmatrix} x_{-1}(kT) \\ x_2(kT) \end{bmatrix}$$

where:

$$\tilde{A} = e^{\Phi_o AT} \tag{20}$$

$$\tilde{B}_1 = \left[\int_0^T e^{\Phi_0 A\tau} d\tau \right] \Phi_0 B \tag{21}$$

$$\tilde{E}_1 = (\Phi_{-1}E - T \times I_n)^{-1}\Phi_{-1}E \tag{22}$$

$$\tilde{B}_2 = T(\Phi_{-1}E - T \times I_n)^{-1}\Phi_{-1}B \tag{23}$$

Based in this, the next algorithm is proposed to find the corresponding discrete system of (1):

Algorithm 1 Given a continuous system in the form (1), find the corresponding discrete system in the form of (19).

Step 1 Check if the pair (E, A) is regular, if $\det(\gamma E - A) \neq 0$, then continue.
Step 2 Find the matrices $(P,\, Q)$ of r.s.e form
Step 3 Compute the Drazin inverse, then find Φ_0 and Φ_{-1}
Step 4 Compute $\Phi_{-k} = -\Phi_{-1}E\Phi_{-k+1} = \Phi_{-1}\left(-E\Phi_{-1}\right)^k$, for $k = 2, 3, ..., h$ and $\Phi_k = \Phi_0\left(A\Phi_0\right)^k = \Phi_0 A\Phi_{k-1},\ k = 1, 2, ...$
Step 5 Compute $\tilde{A},\, \tilde{B}_1,\, \tilde{E}_1,\, \tilde{B}_2$ from (20, 21, 22) and (23) respectively.
Step 6 Rewrite equations in the form of (19)

III. DESCRIPTOR PACKAGE FOR SCILAB

Scilab has a very useful set of commands for the management of system descriptors and pecils. The most important functions are:

function	Description
pencan	compute matrices P and Q
spec	eigenvalues of pencil and matrices
tf2des	transfer function to descriptor
ss2des	transfer function to descriptor
des2tf	descriptor to transfer function conversion
sm2des	system matrix to descriptor
pol2des	polynomial matrix to descriptor form

However, there has not special tools for descriptors system management , such as controllability, observability, stability, discretization and others.

It should be noted that currently there are few works about it. For example, in (Varga, 2000) a toolbox for MATLAB is proposed, however, this work is not available. The other work that was done for Mathematica (Vardulakis *et al.*, 2008).

Due to the lack of adequate tools for descriptors system management, in this work have been scheduled a set of functions that complement the scilab package. This functions are showed in the next table.

Command	Description
funmatrix	compute the Laurent expansion coefficients (12) and (13)
dcontr	computation of C, R and I controllability matrices with tree different methods
dobsv	Computation of C, R and I observability matrices with tree different methods
abcdcoeff	a,b,c and d coefficients for Darouach observer (Darouach y Boutayeb, 1995)
kwrse	Compute the Kronecker-Weistrauss r.s.e
qrrse	Compute the QR r.s.e
invrse	compute the inverse r.s.e form
c2dd	transform continuous to discrete using the algorithm 1

IV. NUMERICAL EXAMPLE

Considering the following descriptor system.

$$
E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

$$
A = \begin{pmatrix} -0.8775 & 0 & 0.526 & -0.0274 \\ -5.85 & -0.5 & 0.1481 & 0.0026 \\ 0 & 0.5 & -1 & 0.2 \\ 0 & 2.6522 & -0.274 & -2 \end{pmatrix}
$$

$$
B = \begin{pmatrix} 0 & 0 \\ -0.0856 & 0.01 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
$$

is easily verifiable that pair (E, A) is regular, then exist two matrices

$$
P = \begin{pmatrix} -0.05 & -1.31 & -0.22 & -0.02 \\ 0.94 & -0.12 & 0.47 & 0.03 \\ 0 & 0 & -0.99 & -0.04 \\ 0 & 0 & -0.03 & 0.70 \end{pmatrix}
$$

$$
Q = \begin{pmatrix} -0.09 & 1.06 & 0 & 0 \\ -0.76 & -0.04 & 0 & 0 \\ -0.57 & -0.03 & 0.98 & -0.08 \\ -0.93 & -0.05 & -0.11 & -0.70 \end{pmatrix}
$$

that permit convert the system into the r.s.e form (7) and (8)

$$
\begin{aligned}
\dot{x}_1(t) &= \begin{pmatrix} -1.09 & 8.13 \\ -0.28 & -0.17 \end{pmatrix} x_1(t) + \begin{pmatrix} 0.11 & -0.04 \\ 0.01 & 0.03 \end{pmatrix} u(t) \\
0 &= x_2(t) + \begin{pmatrix} 0 & -0.04 \\ 0 & 0.70 \end{pmatrix} u(t) \\
y &= \begin{pmatrix} -1.78 & 0.96 \\ -1.49 & -0.08 \\ -0.57 & -0.03 \end{pmatrix} x_1 + \begin{pmatrix} -0.11 & -0.70 \\ 0.87 & -0.78 \\ 0.98 & -0.08 \end{pmatrix} x_2
\end{aligned}
$$

then the controllability can be computed using the function *«dcontr»* and selecting a method (Direct, Laurent or r.s.e approach). Selecting direct approach the controllability C-matrices are [1]:

$$
\mathscr{C}_s = \begin{pmatrix} 1.88 & 0 & -0.53 & 0.03 & 0 & 0 \\ 5.85 & 1.5 & -0.15 & -0.00 & -0.09 & 0.01 \\ 0 & -0.5 & 1 & -0.2 & 0 & 0 \\ 0 & -2.65 & 0.27 & 2 & 0 & 1 \end{pmatrix}
$$

$$
\mathscr{C}_f = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.09 & 0.01 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
$$

[1] Because the main objective with regard to the analysis of controllability, is to show the great potential of «dcontr» function, the mathematical support is not presented, however this can be consulted in the references (Duan, 2010; Koumboulis y Mertzios, 1999; Yip y Sincovec, 1981)

for slow (7) and fast (8) subsystems respectively, then using the criteria presented in (Duan, 2010)

$$
\begin{aligned}
\mathsf{rank}\mathscr{C}_s &= 4 = n \\
\mathsf{rank}\mathscr{C}_f &= 3 \neq n
\end{aligned}
$$

Due to that the rank of Cs is 4 and Cf is 3, the slow subsystem is C-controllable, but the fast subsytem is not, thenn the system is not C-Controllable. However the system is R-Controllable and I-Controllable. All these conclusions are obtained automatically with the function «dcontr» for the three methods mentioned.

Similarly, using the function «dobsv» the C-Observability matrices are:

$$
\mathscr{O}_s = \begin{pmatrix} 1.88 & 0 & -0.53 & 0.03 \\ 5.85 & 1.5 & -0.15 & -0.00 \\ 0 & -0.5 & 1 & -0.2 \\ 0 & -2.65 & 0.27 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
$$

$$
\mathscr{O}_f = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
$$

for slow and fast subsystems respectively, when

$$
\begin{aligned}
\mathsf{rank}\mathscr{O}_s &= 4 = n \\
\mathsf{rank}\mathscr{O}_f &= 4 = n
\end{aligned}
$$

due that slow-subsystem and fast-subsystem are C-Observable, the system is C-Observable and therefore, the system is R and I- Observable.

The stability analysis is not implemented in a function, but this can be done by the LMITOOL or SCIYALMIP toolboxes. While the discretization of the continuous system can be done with the algorithm proposed and implemented in the function «c2dd».

First, for discretized the systems is necessary compute the Laurent expansions coefficients, this can be done with the function «fundmatrix», then

$$\Phi(s) = \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.97 & 0.10 \\ 0 & 0 & -0.13 & 0.49 \end{pmatrix}}_{\Phi_{-1}} s^0$$

$$+ \underbrace{\begin{pmatrix} 1 & 0 & 0.52 & 0.04 \\ 0 & 1 & 0.14 & 0.02 \\ 0 & 0.74 & 0.11 & 0.01 \\ 0 & 1.22 & 0.18 & 0.02 \end{pmatrix}}_{\Phi_0} s^{-1}$$

$$+ \underbrace{\begin{pmatrix} -0.88 & 0.36 & -0.40 & -0.03 \\ -5.85 & -0.39 & -3.07 & -0.23 \\ -4.36 & -0.29 & -2.29 & -0.17 \\ -7.16 & -0.47 & -3.76 & -0.28 \end{pmatrix}}_{\Phi_1} s^{-2}$$

$$+ \underbrace{\begin{pmatrix} -1.33 & -0.45 & -0.75 & -0.06 \\ 7.39 & -1.95 & 3.53 & 0.25 \\ 5.51 & -1.45 & 2.63 & 0.19 \\ 9.05 & -2.38 & 4.32 & 0.31 \end{pmatrix}}_{\Phi_2} s^{-3}$$

considering a sample time $T_s = 0.1s$ the discrete system in the form of (19) is:

$$x_1((k+1)T) = \begin{pmatrix} 0.91 & 0.03 & -0.00 & -0.00 \\ -0.55 & 0.95 & -0.00 & -0.00 \\ -0.41 & -0.04 & 1 & -0.00 \\ -0.67 & -0.06 & 0 & 1 \end{pmatrix} x_1(kT)$$

$$+ \begin{pmatrix} -0.00 & 0.00 \\ -0.01 & 0.00 \\ -0.01 & 0.00 \\ -0.01 & 0.00 \end{pmatrix} u(Tk)$$

$$0 = x_2(kT) + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -0.10 \\ 0 & -0.49 \end{pmatrix} u(kT)$$

V. CONCLUSIONS

In this paper has been presented a set of tools to analyze systems in descriptor form, this tool includes the implementation of different methods to study the controllability and observability of descriptor systems, each of them to determine the C-controllability (observability), R-controllability (observability), the I-controllability (observability) and hence the S-Controllability (observability). Each of these methods is aimed at solving the problem from a very particular case, so that you can work using the Laurent expansion (which do more easy the discretization), in a canonical form or analyzing the system directly. The Laurent expansion approach is used by discretize continuous systems, two methods were analyzed based in the work of Karampetakis and is proposed a algorithm of solution. .

Due that no exist a Descriptor Toolbox available for Scilab, a set of functions have been written with the intention to create a Descriptor Scilab toolbox available to the community.

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